APPROXIMATE DETERMINATION OF THE SHOCK-ADIABAT EXPONENT FOR MOTION OF LARGE METEORIC BODIES IN THE ATMOSPHERE

A. K. Stanyukovich

Zhurnal Prikladnoi Mekhaniki i Tekhnicheskoi Fiziki, Vol. 10, No. 2, pp. 104–105, 1969

An approximate method for finding the parameters of the shock wave which forms during motion of a meteoric body in the atmosphere at high supersonic speed is described via the conventional gasdynamic formulas after preliminary determination of the effective adiabatic exponent. The energy losses in dissociation and ionization of the air are accounted for.

The basic equation [1] for the normal compression shock, under the assumption that the energy losses have a purely thermal nature, in a system fixed with the traveling body, has the form

$$E_1 - E_2 = {}^1/_2\,(p_2 + p_1)\,(V_1 - V_2) + Q. \qquad (1)$$

Here E_1, V_1, and p_1 are, respectively, the internal energy, specific volume, and pressure ahead of the shock wave front; E_2, V_2, and p_2 are the same quantities behind the shock wave front; Q are the energy losses in dissociation and ionization of the air.

For high supersonic velocities we can neglect the quantities E_1 and p_1. Assuming that $E_2 = C_v T_2 = p_2 V_2 / (k - 1)$, we obtain an equation of the form

$$\frac{p_2 V_2}{k - 1} = \frac{p_2}{2}(V_1 - V_2) + Q, \qquad (2)$$

where K = 1.41 is the adiabatic exponent for the air ahead of the shock wave.

We introduce the dimensionless quantity $q = -2Q/p_2V_2$. Substituting q into (2) and solving for $V_1/V_2 = \rho_2^*/\rho_1$, we obtain

$$\frac{\rho_2^*}{\rho_1} = \frac{k+1}{k-1} + q. \qquad (3)$$

Here ρ_2^* is the density behind the shock wave in the presence of dissociation and ionization. The quantity $(k + 1)/(k - 1)$ is equal to the maximum shock compression in the absence of dissociation and ionization processes and for $M_1 \to \infty$; therefore q denotes the increase of the shock compression in comparison with the maximum compression resulting from dissociation and ionization of the air, i.e.,

$$q = \frac{\rho_2^*}{\rho_1} - \frac{\rho_2^{\max}}{\rho_1} = \frac{\Delta\rho_2^*}{\rho_1}.$$

We introduce the effective adiabatic exponent k*. By analogy with the known aerodynamic formula we take

$$\frac{\rho_2^*}{\rho_1} = \frac{(k^* + 1)\,M_1^2}{2 + (k^* - 1)\,M_1^2}. \qquad (4)$$

It is not difficult to see that in this case the adiabatic exponent is not constant, but depends on M_1. Replacing ρ_2^*/ρ_1 in (3) by (4) and solving for k*, we find

$$k^* = \frac{2k + q\,(k - 1)(1 - 2/M_1^2)}{2 + q\,(k - 1)}. \qquad (5)$$

In practice, for determining the effective adiabatic exponent we can use the Rozhdestvenskii curves [2], which show the variation of the shock compression ρ_2^*/ρ_1 as a function of Mach number for different pressures p_1 of the unperturbed flow. Substituting ρ_2^*/ρ_1 into (4) and solving for k*, we find

$$k^* = \frac{1}{\rho_2^*/\rho_1 - 1}\left[\frac{\rho_2^*}{\rho_1}\left(1 + \frac{2}{M_1^2}\right) + 1\right]. \tag{6}$$

For high supersonic speeds (in practice for $M_1 > 30$) we can neglect the term $2/M_1^2$. Then k* is found from the equation

$$k^* = \frac{\rho_2^*/\rho_1 + 1}{\rho_2^*/\rho_1 - 1}. \tag{7}$$

It is usual to consider that shock wave formation ahead of a meteoric body begins at an altitude of 80–100 km [3]. This altitude corresponds to a pressure $p_1 \sim 10^{-4}$ atm abs. We present the results of a calculation made for this pressure and four values of the Mach number using (6) and the curves of [2]

$M_1 =$	25	30	35	40,
$\rho_2^*/\rho_1 =$	18.5	17.8	18.5	19.9,
$k^* =$	1.130	1.120	1.116	1.111.

Bronshtén [3, 4] recommends for such cases values of k* lying in the range 1.11–1.29. We note that accounting for recombination of the air molecules may increase k* slightly.

REFERENCES

1. F. A. Baum, S. A. Kaplan, and K. P. Stanyukovich, Introduction to Cosmic Gasdynamics [in Russian], Fizmatgiz, Moscow, 1958.
2. I. B. Rozhdestvenskii, "Thermodynamic and gasdynamic properties of air flow behind a normal compression shock with account for dissociation and ionization of the air," collection: Physical Gasdynamics [in Russian], Izd-vo AN SSSR, Moscow, 1959.
3. V. A. Bronshtén, "Problems of the motion of large meteorites," Meteoritika, Nauka, Moscow, no. 24, 1964.
4. V. A. Bronshtén, "Theory and spectral study of meteorite-forming bolides," Meteoritika, Nauka, Moscow, no. 27, 1966.

1 December 1968

Moscow